AP® CALCULUS BC
2016 SCORING GUIDELINES

Question 2

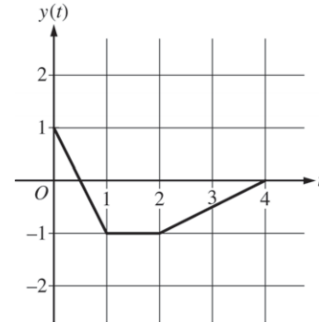
At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.



(a) $x(3) = x(0) + \int_0^3 (t^2 + \sin 3t^2)\, dt = 14.377$

$y(3) = -\frac{1}{2}$

$(x(3), y(3)) = (14.377, -0.5)$

2 : { 1 : expression for $x(3)$; 1 : answer }

(b) $\frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{dy}{dt}$

$\left.\frac{dx}{dt}\right|_{t=3} = 3^2 + \sin(3 \cdot 3^2) = 9.956$

$\left.\frac{dy}{dt}\right|_{t=3} = \frac{1}{2}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 0.050$

2 : { 1 : chain rule with respect to x; 1 : answer }

(c) Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{9.956^2 + \left(\frac{1}{2}\right)^2} = 9.969$

2 : { 1 : expression for speed; 1 : answer }

(d) $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^1 \sqrt{(t^2 + \sin 3t^2)^2 + (-2)^2}\, dt + \int_1^2 \sqrt{(t^2 + \sin 3t^2)^2 + (0)^2}\, dt$

$= 4.350$

3 : { 1 : expression for arc length; 1 : splits up integral; 1 : answer }