6. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.

   (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

   (b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

   (c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

   (d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

a. $f^{(2)}(1) = (-1)^2 \frac{(2-1)}{2^2} = \frac{1}{4}$

$f^{(3)}(1) = (-1)^3 \frac{(3-1)!}{2^3} = -\frac{1}{4}$

$p_4(1) = 1 - \frac{1}{2}(x-1) + \frac{(x-1)^2}{8} - \frac{(x-1)^3}{24} + \cdots + \frac{(-1)^n (x-1)^n}{2^n \cdot n}$

2 point: $\begin{cases} 1: first\ four\ nonzero\ term \\ 1: general\ term \end{cases}$

b. The series centered at $x = 1$, radius of convergence is 2: $|x - 1| < 2 \rightarrow -1 < x < 3$

When $x = 3$: $\sum_{n=2}^{\infty} \frac{(-1)^n (x-1)^n}{2^n \cdot n}$

$= \sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{2^n \cdot n} = \sum_{n=2}^{\infty} \left(\frac{(-1)^n}{n}\right)$ *converge*

When $x = -1$: $\sum_{n=2}^{\infty} \frac{(-1)^n (x-1)^n}{2^n \cdot n} = \sum_{n=2}^{\infty} \frac{(-1)^n (-2)^n}{2^n \cdot n}$

$= \sum_{n=2}^{\infty} \left(\frac{1}{n}\right)$ *diverge*

The interval of convergence: $-1 < x < 3$

2 points: 1: find the end point, state if converge or diverge

1: interval of convergence

c. f (1.2)= $1 - \frac{(1.2-1)}{2} + \frac{(1.2-1)^2}{2 \cdot 2^2} = \frac{181}{200}$	1 point: writing out three terms 1 point: writing out $\frac{181}{200}$
d. $\lvert E_n(x)\rvert = \lvert f(x) - P_n(x)\rvert < \frac{1}{1000}$ Because 1.2 lies in the interval of convergence $-1 \leq x \leq 3$, so by the Alternating Series Estimation Theorem, the approximation differs from the exact value by less than the absolute value of the fourth term. The fourth term: $\frac{\lvert(-1)^3(1.2-1)^3\rvert}{2^3 \cdot 3} = \frac{1}{3000}$ Because: $\frac{1}{3000} < \frac{1}{1000}$ Therefore: the approximation in part (c) in within 0.0001 of the exact value of f(1.2)	1 point: Stating the Alternating Series 2 points: calculating the fourth term and state $\frac{1}{3000} < \frac{1}{1000}$