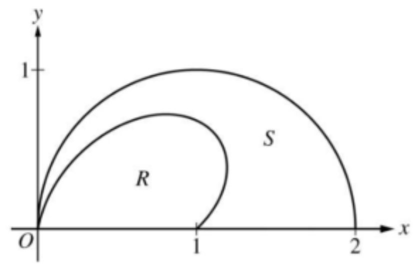
Question 2



2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x-axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x-axis.

(a) Find the area of R.

(b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

(c) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

(d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

(a) $R = 1/2 \int_0^{\pi/2} f(\theta)^2 d\theta$ ≈ 0.648	3 : { 1: limit, constant 1: integrand 1: answer
(b) $\int_0^k (g(\theta) - f(\theta))^2 \, d\theta = \int_k^{\frac{\pi}{2}} (g(\theta) - f(\theta))^2 \, d\theta$	2 : { 1: limit 1: integrand

(c) $w(\theta) = g(\theta) - f(\theta)$

$w_A = \frac{1}{\frac{\pi}{2}} \int_0^{\pi/2} w(\theta)\, d\theta \approx 0.485$

2 : { 1: expression of $w(\theta)$
1: expression of w_A, answer

(c) $w(\theta) = g(\theta) - f(\theta) = 0.485$

$\theta \approx 0.581$

$w'(0.581) \approx -0.582$

Since $w'(\theta) < 0$, $w(\theta)$ is decreasing at $\theta = 0.581$

2 : { 1: value of θ
1: answer and explanation