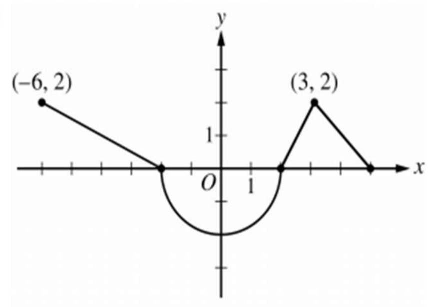
2017 AP Calculus BC Scoring Guidelines

Question 3

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.



Graph of f'

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

(a) $f(-6) = f(-2) - \int_{-2}^{-6} f'(x)dx$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)dx$

$f(-6) = 3$

$f(5) = 10 - 2\pi$

2: { 1 Integrals; 1 Answers

(b) $f(x)$ increases on intervals $2 < x < 5$ and $-6 < x < -2$ because $f'(x)$ is positive on those intervals.

2: { 1 Intervals; 1 Justification

(c) The absolute minimum value of a function occurs either at an endpoint of the function or a point where $f'(x) = 0$. The endpoints exist at $x = -6$ and $x = 5$ and $f'(x) = 0$ at $x = -2$ and $x = 2$.

x	$f(x)$
6	3
5	$10 - 2\pi$
-2	7
2	$7 - 2\pi$

On the interval $[-6,5]$ the absolute minimum value is $7 - 2\pi$ at $f(2)$.

2: { 1 Absolute Minimum; 1 Justification

(d) $f''(-5) = -\frac{1}{2}$ = slope of $f'(x)$ at $x = -5$

$f''(3) = DNE$ because a jump discontinuity exists on the graph of $f'(x)$ at $x = 3$

$\lim_{x\to3^+} f(x) \neq \lim_{x\to3^-} f(x)$

3: { 1 Answer for $f''(-5)$; 1 Answer for $f''(3)$; 1 Justification for $f''(3)$