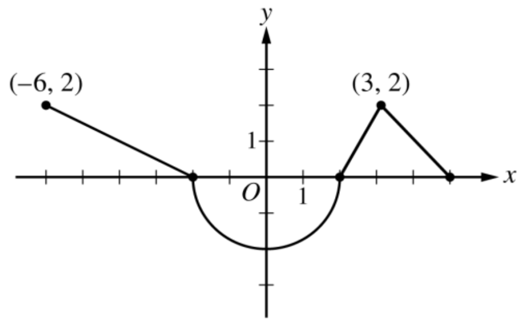
AP® CALCULUS BC
2017 SOCRING GUILDELINES



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find the values of $f(-6)$ and $f(5)$.

   (b) On what intervals is f increasing? Justify your answer.

   (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

   (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

(a) $f(-6) = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - (\frac{1}{2})(4 \cdot 2) = 3$

$f(5) = 7 + \int_{-2}^{5} f'(x)\,dx = 7 - \frac{1}{2}\pi(2)^2 + (\frac{1}{2})(3 \cdot 2) = 10 - 2\pi$

$2 : \begin{cases} 1 : f(-6) \text{ answer} \\ 1 : f(5) \text{ answer} \end{cases}$

(b) The graph of f is increasing on the intervals $-6 < x < -2$ and $2 < x < 5$ because f' is positive on these intervals.

$2 : \begin{cases} 1 : \text{two intervals} \\ 1 : \text{justification} \end{cases}$

(c) The absolute minimum value of f could occur when $f'(x) = 0$ at $x = 2$ or at the endpoints

$f(-6) = 3$

$f(2) = f(-2) + \int_{-2}^{2} f'(x)\,dx = 7 - \frac{1}{2}\pi(2)^2 = 7 - 2\pi$

$f(5) = 10 - 2\pi$

$f(2) < f(-6) < f(5)$

Therefore, the absolute minimum occurs at x=2 and $f(2) = 7 - 2\pi$

$3 : \begin{cases} 1 : \text{consider all possible candidates} \\ 1 : \text{find and compare candidates} \\ 1 : \text{answer with justification} \end{cases}$

(d) $f''(-5)$ = slope at x=5 of $f' = -\frac{1}{2}$

$f''(3)$ does not exist because the graph of f' is not differentiable at x = 3 because there is a corner.

$2 : \begin{cases} 1 : f''(-6) \text{ answer} \\ 1 : f''(5) \text{ answer with justification} \end{cases}$