2017 AP® CALCULUS BC FREE-RESPONSE QUESTIONS
SCORING GUIDELINES
Question 4

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

   (a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

   (b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

   (c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

(a) $H - H_0 = m(T - T_0)$

$H_0 = 91$

$T_0 = 0$

$m = \frac{dH}{dT}\Big|_{(T,H)=(0,91)} = -\frac{1}{4}(91 - 27) = -16$

An equation for the line tangent to (0,91) is $H(T) = 91 - 16T$

$H(3) \approx 91 - (16 \cdot 3) = 43$°C

2: { 1: tangent line equation using $\frac{dH}{dT}\Big|_{(0,91)}$; 1: answer (local linearity approximation) }

(b) $\frac{d^2H}{dT^2} = \frac{d}{dT}\left(-\frac{1}{4}(H - 27)\right) = -\frac{1}{4}$ for all $t > 0$

$\frac{d^2H}{dT^2} < 0$ on the interval $0 < t < 3$

The answer in part (a) is, therefore, an overestimate since H(t) will be concave down on $0 < t < 3$.

2: { 1: $\frac{d^2H}{dT^2}$ at t=0; 1: answer with reason }

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(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

(c) $(G - 27)^{-\frac{2}{3}}\, dG = -dT$

$\int (G - 27)^{-\frac{2}{3}}\, dG = -\int dT$

$3(G - 27)^{\frac{1}{3}} = -T + C$

$3(91 - 27)^{\frac{1}{3}} = 0 + C \Longrightarrow C = 12$

$3(G - 27)^{\frac{1}{3}} = -T + 12$

$(G - 27)^{\frac{1}{3}} = -\frac{T}{3} + 4$

$((G - 27)^{\frac{1}{3}})^3 = (-\frac{T}{3} + 4)^3$

$G(T) = \left(-\frac{T}{3} + 4\right)^3 + 27$ for all times $T < 10$

$G(3) = \left(-\frac{3}{3} + 4\right)^3 + 27 = 54$°C

5:
- 1: separation of variables
- 1: antiderivatives
- 1: constant of integration
- 1: uses initial condition
- 1: solves for G

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables