AP® CALCULUS BC
2016 SCORING GUIDELINES

Question 5

5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

   (a) Find the average value of the radius of the funnel.

   (b) Find the volume of the funnel.

   (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

(a) $\dfrac{\frac{1}{20}(3+0^2)+\frac{1}{20}(3+10^2)}{2} \approx 2.65$ inches

$2 : \begin{cases} 1 : \text{Equation for the average} \\ 1 : \text{Answer} \end{cases}$

(b) $\pi\int_0^{10} r(h)^2\,dh$ is the total volume of the funnel between the height $0 \le h \le 10$ inches.

$\pi\int_0^{10}\left(\frac{1}{20}(3+h)\right)^2 dh = 173.494$ Inches^2

$3 : \begin{cases} 1 : \text{Set up correct area equation} \\ 1 : \text{Plug in for } r \text{ and a correct interval} \\ 1 : \text{Answer} \end{cases}$

(c) $r = \dfrac{3}{20} + \dfrac{h^2}{20}$

$\dfrac{d}{dh}\left(r = \dfrac{3}{20} + \dfrac{h^2}{20}\right) =$

$dr = \dfrac{h}{10}\,dh$

$\dfrac{1}{5} = \dfrac{3}{10}\,dh$

$\dfrac{2}{3} = dh$

$4 : \begin{cases} 1 : \text{Differentiation equation} \\ 1 : \text{Correct dr} \\ 1 : \text{Plug in right values} \\ 1 : \text{Answer} \end{cases}$