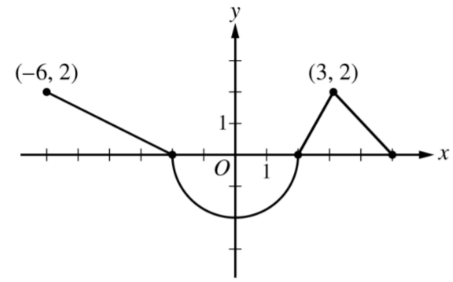
Question 3

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.



Graph of f'

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

(a) $f(-6) = f(-2) - \int_{-6}^{-2} f'(x)dx$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)dx$

$f(-6) = 3$

$f(5) = 10 - 2\pi$

3 : { 1: proper integrals; 2: both answers }

(b) $f(x)$ is increasing on the intervals $-6 < x < -2$ and $2 < x < 5$ because $f'(x)$ is positive on these intervals.

2 : { 1: justification; 1: intervals }

(c) $f'(x)$ changes sign at $x = -2$ and $x = 2$

x	$f(x)$
-6	3
-2	7
2	7-2π
5	10-2π

The endpoints should be examined for justification

On the closed interval [-6,5] the absolute minimum value is $f(2) = 7 - 2\pi$.

2 : { 1: justification – with the candidates test, or a logical explanation based on the graph; 1: x and $f(x)$ values }

(d) $f''(-5) = -\frac{1}{2}$: slope at $x = -5$

$f''(3)$: does not exist

$\lim_{x\to3^+}(f''(x)) \neq \lim_{x\to3^-}(f''(x))$

2 : { 1: answer for $f''(-5)$; 1: valid explanation for $f''(3)$ }