AP® CALCULUS BC
2017 SCORING GUIDELINES

Question 4

At time t = 0, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time t = 0, and the internal temperature of the potato is greater than 27°C for all times t > 0. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at t = 0. Use this equation to approximate the internal temperature of the potato at time t = 3

(b)Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time t = 3.

(c) For t < 10, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where G(t) is measured in degrees Celsius and G(0) = 91. Find an expression for G(t). Based on this model, what is the internal temperature of the potato at time t = 3?

(a) $\frac{dH}{dt} = -\frac{1}{4}\big((91) - 27\big) = -16$
The tangent line is
$H = -16t + 91$
$H(3) \approx -48 + 91 \approx 43$°C

2: $\begin{cases} 1: Tangent\ line \\ 1: answer \end{cases}$

(b) $\frac{d^2H}{dt^2} = \frac{d}{dt}\left(\frac{dH}{dt}\right) = \frac{1}{16}(H - 27)$
Therefore $\frac{d^2H}{dt^2} > 0$ on the interval $0 \le t \le 3$ so the answer from part (a) is an underestimate.

2: $\begin{cases} 1: \frac{d^2H}{dt^2} \\ 1: answer\ with\ justification \end{cases}$

(c) $\frac{dG}{dt} = -(G - 27)^{\frac{2}{3}}$

$\int \frac{1}{(G-27)^{2/3}}\, dG = \int -\, dt$

$3(G - 27)^{1/3} = -t + C$
$3((91) - 27)^{1/3} = -(0) + C \Rightarrow C = 12$
$3(G - 27)^{1/3} = -t + 12$
$G(t) = \left(-\frac{t}{3} + 4\right)^3 + 27$
$G(3) = \left(-\frac{3}{3} + 4\right)^3 + 27 = 54$°C

5: $\begin{cases} 1: seperation\ of\ variables \\ 1: antiderivatives \\ 1: constant\ of\ integration \\ 1: uses\ initial\ condition \\ 1: anwser \end{cases}$