# AP® CALCULUS AB/CALCULUS BC
# 2017 SCORING GUIDELINES

## Question 5

Let $f$ be the function defined by $f(x) = \dfrac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of $f$ at $x = 3$.

(b) Find the $x$-coordinate of each critical point of $f$ in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that $\dfrac{3}{2x^2 - 7x + 5} = \dfrac{2}{2x - 5} - \dfrac{1}{x - 1}$, evaluate $\int_5^\infty f(x)\,dx$ or show that the integral diverges.

(d) Determine whether the series $\sum_{n=5}^{\infty} \dfrac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

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(a) $f'(x) = -3\left(2x^2 - 7x + 5\right)^{-2}(4x - 7)$

$f'(3) = -3\left(2(3)^2 - 7(3) + 5\right)^{-2}(4(3) - 7) = \dfrac{-15}{4}$

2 : { 1 : derivative; 1 : answer }

(b) $f'(x) = 0$ at $x = \dfrac{7}{4}$

Therefore, f has a critical point at $x = \dfrac{7}{4}$

$f'(x) > 0$ for $x < \dfrac{7}{4}$ and $f'(x) < 0$ for $x > \dfrac{7}{4}$

Therefore, f has a relative maximum at $x = \dfrac{7}{4}$

3 : { 1 : critical point at $x = \dfrac{7}{4}$; 1 : answer; 1 : justification }

(c) $\displaystyle\int_5^\infty f(x)\,dx = \int_5^\infty \frac{2}{2x^2 - 5} - \frac{1}{x - 1}\,dx$

$\displaystyle\lim_{b\to\infty} \int_5^b \left(\frac{2}{2x^2 - 5} - \frac{1}{x - 1}\right)dx = \lim_{b\to\infty} \Big[\ln|2x - 5| - \ln|x - 1|\Big]_5^b$

$\displaystyle= \lim_{b\to\infty} \left[\ln\left(\frac{2x - 5}{x - 1}\right)\right]_5^b$

$\displaystyle= \lim_{b\to\infty} \left(\ln\left(\frac{2b - 5}{b - 1}\right)\right) - \ln\left(\frac{5}{4}\right)$

$\displaystyle= \ln(2) - \ln\left(\frac{5}{4}\right) = \ln\left(\frac{8}{5}\right)$ or $\displaystyle -\ln\left(\frac{5}{8}\right)$

2 : { 1 : antiderivative; 1 : answer }

(d) f is continuous, positive, and decreasing on $[5, \infty)$

$\displaystyle\int_5^\infty f(x)\,dx$ converges to $\displaystyle\ln\left(\frac{8}{5}\right)$

Therefore, $\displaystyle\sum_{n=5}^{\infty} f(n)$ converges by integral test

2 : { 1 : conditions; 1 : answer }