AP® CALCULUS AB/CALCULUS BC
2017 SCORING GUIDELINES

$$f(0) = 0$$
$$f'(0) = 1$$
$$f^{(n+1)}(0) = -n \cdot f^{(n)}(0) \text{ for all } n \geq 1$$

6. A function f has derivatives of all orders for $-1 < x < 1$. The derivatives of f satisfy the conditions above. The Maclaurin series for f converges to $f(x)$ for $|x| < 1$.

   (a) Show that the first four nonzero terms of the Maclaurin series for f are $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$, and write the general term of the Maclaurin series for f.

   (b) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.

   (c) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t)\,dt$.

   (d) Let $P_n\left(\frac{1}{2}\right)$ represent the nth-degree Taylor polynomial for g about $x = 0$ evaluated at $x = \frac{1}{2}$, where g is the function defined in part (c). Use the alternating series error bound to show that

   $$\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| < \frac{1}{500}.$$

(a) Let $P(x)$ denote the Maclaurin series for f $f^{(n)}(0) = (-1)^{n-1}(n-1)!$ $P(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{3} + \cdots + \frac{f^{(n)}(0)x^n}{n!} + \cdots$ $= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^{n-1}x^n}{n} + \cdots$	2: $\begin{cases} 1: f^{(n)}(0) \\ 2: \text{first four and general term} \end{cases}$
(b) $P(1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converges by the Alternating Series Test. Let $Q(1)$ be the series of absolute values of each term of $P(1)$ $P(1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ diverges because it's the Harmonic series. Thus, the Maclaurin series converges conditionally.	3: $\begin{cases} 1: \textit{analyze } P(x) \\ 2: \textit{analyze } Q(x) \\ 3: \text{convergence} \end{cases}$

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(b) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.

(c) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t)\,dt$.

(d) Let $P_n\left(\frac{1}{2}\right)$ represent the nth-degree Taylor polynomial for g about $x = 0$ evaluated at $x = \frac{1}{2}$, where g is the function defined in part (c). Use the alternating series error bound to show that

$$\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| < \frac{1}{500}.$$

(c) $$g(x) = \int_0^x f(t)\,dt = \int_0^x \left(t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \cdots + \frac{(-1)^{n-1}t^n}{n} + \cdots \right) dt$$
$$= \left(\frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{12} - \frac{t^5}{20} + \cdots + \frac{(-1)^{n-1}t^{n+1}}{n(n+1)} + \cdots \right) \Bigg|_0^x$$
$$= \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20} + \cdots + \frac{(-1)^{n-1}x^{n+1}}{n(n+1)} + \cdots$$

2: { 1: first four terms
1: general term

(d) $$\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| \leq \frac{\left(\frac{1}{2}\right)^5}{20} = \frac{1}{640} < \frac{1}{500}$$

2: { 1: uses the fourth term
1: error bound