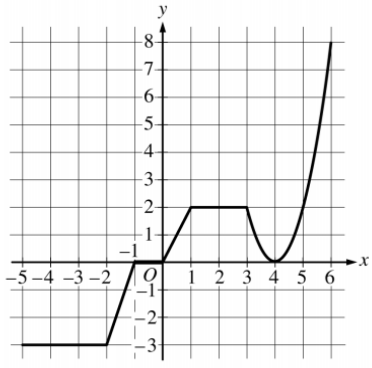
AP® CALCULUS BC
2018 SCORING GUIDELINES

Question 3



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

(a) $f(1) = 3$

$$f(-5) = f(1) + \int_1^{-5} f'(x)\,dx$$

$$= f(1) - \int_{-5}^{1} g(x)\,dx$$

$$= 3 - \left(-3(3) - \frac{1}{2}(1)(3) + \frac{1}{2}(1)(2)\right)$$

$$= \frac{25}{2}$$

$2 : \begin{cases} 1 : f(-5) \\ 1 : \text{answer} \end{cases}$

(b) $$\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$$

$$= 4 + \int_3^6 2(x-4)^2\,dx$$

$$= 4 + \left(\frac{2x^3}{3} - \frac{16x^3}{3} + 32x \,\Big|_1^6\right)$$

$$= 4 + 6 = 10$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $f'(x) = g(x)$

The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$, because $f'(x) = g(x)$ is positive and increasing on these intervals.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ since $f'(x) = g(x)$ is decreasing for $3 < x < 4$ and increasing for $4 < x < 6$.

1 : answer with justification

By Anh & Fuka @Williston Northampton School