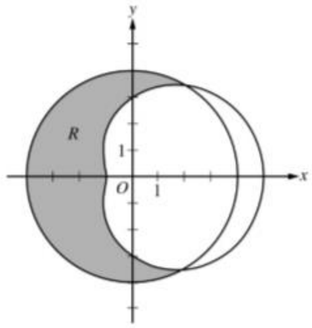
AP CALCULUS BC
2018 SCORING GUIDELINES

QUESTION 5

The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.



(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

(a) $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (4^2 - (3 + 2\cos\theta)^2)\, d\theta$

1 : equation

(b) $\frac{32\pi}{3} - \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (3 + 2cos\theta)^2\, d\theta$

$$\frac{dy}{dx} = \frac{dy/\theta\,\big|_{\theta=\frac{\pi}{2}}}{dx/\theta\,\big|_{\theta=\frac{\pi}{2}}} = \frac{-2}{-3} = \frac{2}{3}$$

$4 : \begin{cases} 1 : \text{ integrand} \\ 1 : dy/dx = (dy/\theta)/(dx/\theta) \\ 1 : \textit{ limits and constant} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dr}{dt} = 3$

$$3 = -2\left(\frac{\sqrt{3}}{2}\right)\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{3}{\sqrt{3}} = \sqrt{3}\; radian/second$$

$4 : \begin{cases} 1 : dr/dt = 3 \\ 2 : \ d\theta/dt \\ 1 : \textit{answer with units} \end{cases}$