MATHEMATICAL ASSOCIATION OF AMERICA

Solutions Pamphlet

American Mathematics Competitions

28th Annual

AMC 8

American Mathematics Contest 8

Tuesday, November 13, 2012

This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students. However, the publication, reproduction, or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. *Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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American Mathematics Competitions

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1. **Answer (E):** Rachelle needs $\frac{24}{8} = 3$ times the amount of meat for the picnic than she would use for her family. So she needs $3 \times 3 = 9$ pounds of meat.

   OR

   Set up a proportion to compare the two ratios of pounds of meat to number of hamburgers.
   $$\frac{3}{8} = \frac{x}{24}$$
   Solving for x, $8x = 72$, so $x = 9$ pounds of meat.

2. **Answer (B):** The net growth per day in East Westmore is 3 births − 1 death = 2 people. There are typically 365 days in a year, so the population grows by about $2 \times 365 = 730$, or close to 700 people a year.

3. **Answer (B):** From 6:57 AM to 12:00 PM (noon) is 5 hours and 3 minutes. Since the length of daylight is 10 hours and 24 minutes, there must be another 5 hours and 21 minutes until sunset. The correct sunset time is 5:21 PM.

4. **Answer (C):** The whole slice that Peter ate was $\frac{1}{12}$ of the pizza. His half of the second slice was half of $\frac{1}{12}$, or $\frac{1}{24}$, of the pizza. The fraction of the pizza that Peter ate was
   $$\frac{1}{12} + \frac{1}{24} = \frac{2}{24} + \frac{1}{24} = \frac{3}{24} = \frac{1}{8}.$$

5. **Answer (E):** The vertical sides on the left add up to $5 + X$ while the vertical sides on the right add up to 10. Therefore $X = 5$.

6. **Answer (E):** The width of the frame is $10 + 2 + 2 = 14$ inches, and its height is $8 + 2 + 2 = 12$ inches. It encloses an area of $14 \times 12 = 168$ square inches. The photograph occupies $10 \times 8 = 80$ square inches of that area, so the area of the border itself is $168 - 80 = 88$ square inches.

7. **Answer (B):** To achieve an average grade of 95 on the four tests, Isabella must score a total of $4 \times 95 = 380$ points. She scored a total of $97 + 91 = 188$ points on her first two tests, so she must score a total of at least $380 - 188 = 192$ points on her last two tests. Because she can score at most 100 on her fourth test, she must have scored at least 92 on her third test.

   OR

   Isabella's first score was $95 + 2$, her second score was $95 - 4$, and her fourth score can be at most $95 + 5$. Because she can still achieve an average score of 95, her third score must have been at least $95 - 2 + 4 - 5 = 92$.

8. **Answer (D):** The price of an item costing $\$d$ after both discounts are applied is $.80(.50d) = .40d$, a discount of 60% off the original price.

9. **Answer (C):** All 200 heads belonged to animals with at least two legs, accounting for 400 of the 522 legs. The additional 122 legs belonged to four-legged mammals, each of which had two additional legs. So Margie saw $\frac{122}{2} = 61$ four-legged mammals and $200 - 61 = 139$ birds.

10. **Answer (D):** To form a number greater than 1000, the first digit must be 1 or 2. If the first digit is a 1, the remaining numbers could be 022, 202, or 220. If the first digit is a 2, the remaining numbers could be 012, 021, 102, 120, 201, or 210. So there are $3 + 6 = 9$ ways to form a number greater than 1000.

11. **Answer (D):** Because the mode is unique, it must be 6, so the mean must also be 6. The sum of the seven numbers is $31 + x$, which must be equal to $7 \times 6 = 42$. Therefore $x = 11$. The median of the numbers 3, 4, 5, 6, 6, 7, 11 is also 6.

12. **Answer (A):** To determine the units digit of 13^{2012}, looking for patterns is a good approach to finding the solution. The units digit of each power of 13 depends only on the units digit of the previous power, as follows:
For 13^1, the units digit is 3.
For 13^2, $3 \times 3 = 9$, so the units digit is 9.
For 13^3, $9 \times 3 = 27$, so the units digits is 7.
For 13^4, $7 \times 3 = 21$, so the units digit is 1.
For 13^5, $1 \times 3 = 3$, so the units digit is 3.
The units digits of successive powers of 13 follow the pattern 3, 9, 7, 1, 3, 9, 7, 1, ...
Since $2012 = 4 \times 503$, the units digit of 13^{2012} is 1.

13. **Answer (C):** Since $143 = 11 \times 13$ and $187 = 11 \times 17$, the pencils cost 11 cents each. That means Sharona bought $17 - 13 = 4$ pencils more than Jamar.

14. **Answer (B):** If there were 2 teams in the conference, then there would be only 1 game played. Each additional team plays a game against each previously listed team.

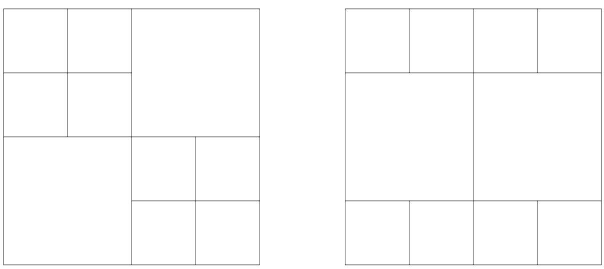
Number of teams	Number of games
2	1
3	$1 + 2 = 3$
4	$3 + 3 = 6$
5	$6 + 4 = 10$
6	$10 + 5 = 15$
7	$15 + 6 = 21$

If 21 conference games were played, then there were 7 teams in the conference.

15. **Answer (D):** Two less than the number must be divisible by 3, 4, 5, and 6. The least common multiple of these numbers is 60, so 62 is the smallest number greater than 2 to leave a remainder of 2 when divided by 3, 4, 5, and 6. Therefore the number lies between 61 and 65.

16. **Answer (C):** The sum will be as large as possible when the largest digits are placed in the most significant places. The 8 and 9 should be in the ten-thousands place, the 6 and 7 in the thousands place, the 4 and 5 in the hundreds place, the 2 and 3 in the tens place, and the 0 and 1 in the units place. The only choice that fits that description is 87431. (In this case the other number is 96520, giving the largest sum of 183951.)

17. **Answer (B):** The area of the original square is a square number that is more than 8, so 16 is the least possible value for the area of the original square. Its side has length 4. Two possible ways of cutting the square are shown below:



18. **Answer (A):** Since the integer is neither prime nor square, it is divisible either by two distinct primes or by the cube of a prime. The smallest prime numbers not less than 50 are 53 and 59. Since $53 \times 59 = 3127 < 53^3$, the smallest number satisfying this description is 3127.

19. **Answer (C):** There are 4 non-blue marbles. That is, there are altogether 4 red and green marbles. There are also 6 non-red marbles and 8 non-green marbles, so there are two more red marbles than green marbles. Therefore there are 3 red marbles and 1 green marble. Because 3 of the 8 non-green marbles are red, the other 5 must be blue. The total number of marbles is $1 + 3 + 5 = 9$.

OR

Let g, b, and r be the number of green, blue, and red marbles respectively. Then

$$g + b = 6$$

$$r + b = 8$$
$$r + g = 4$$

Adding all three equations together, $2g + 2b + 2r = 18$, so $g + b + r = 9$.

20. **Answer (B):** Using a common denominator, $\frac{5}{19} = \frac{105}{399}$ and $\frac{7}{21} = \frac{133}{399}$, so $\frac{5}{19} < \frac{7}{21}$.
Also $\frac{7}{21} = \frac{161}{483}$ and $\frac{9}{23} = \frac{189}{483}$, so $\frac{7}{21} < \frac{9}{23}$.

OR

Comparing each fraction, $\frac{7}{21} = \frac{1}{3}$, $\frac{5}{19} < \frac{5}{15} = \frac{1}{3}$, and $\frac{9}{23} > \frac{9}{27} = \frac{1}{3}$, so the correct increasing order is $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$.

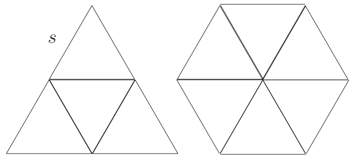
21. **Answer (D):** The surface area of the cube is $6 \times 10^2 = 600$ square feet. The green paint covers 300 square feet, so the total area of the white squares is $600 - 300 = 300$ square feet. There are 6 white squares, so each has area $\frac{300}{6} = 50$ square feet.

22. **Answer (D):** If nine distinct integers are written in increasing order, the median, which is the middle number, would be in the fifth place. If all of the remaining integers are greater than 9, then the median is 9. If all of the remaining integers are less than 3, then the median is 3. All the integers from 3 to 9 are possible medians, as shown in the following table.

Include in R	Median of R
-1, 0, 1	3
0, 1, 5	4
1, 5, 7	5
5, 7, 8	6
7, 8, 10	7
8, 10, 11	8
10, 11, 12	9

So, 3, 4, 5, 6, 7, 8, and 9 are the possible medians, and their count is 7.

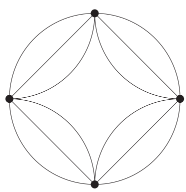
23. **Answer (C):** The equilateral triangle can be divided into 4 smaller congruent equilateral triangles, each with area of 1 and side length s, as shown. Then the original equilateral triangle has perimeter $6s$. A hexagon with perimeter $6s$ can be divided into 6 congruent equilateral triangles, each with area of 1. Therefore the area of the hexagon is 6.



24. **Answer (A):** Translate the star into the circle so that the points of the star coincide with the points on the circle. Construct four segments connecting the consecutive points of the circle and the star, creating a square concentric to the circle.

The area of the circle is $\pi(2)^2 = 4\pi$. The square is made up of four congruent right triangles with area $\frac{1}{2}(2 \times 2) = 2$, so the area of the square is $4 \times 2 = 8$. The area inside the circle but outside the square is $4\pi - 8$.

This is also the area inside the square but outside the star. So, the area of the star is $8 - (4\pi - 8) = 16 - 4\pi$. The ratio of the area of the star figure to the area of the original circle is $\frac{16-4\pi}{4\pi} = \frac{4-\pi}{\pi}$.



25. **Answer (C):** The area of the region inside the larger square and outside the smaller square has total area $5-4 = 1$ and is equal to the area of four congruent right triangles, each with one side of length a and the other of length b. The area of each triangle is $\frac{1}{4}$. If $\frac{1}{2}ab = \frac{1}{4}$, then $ab = \frac{1}{2}$.

The problems and solutions in this contest were proposed by Bernardo Abrego, Sam Baethge, Betsy Bennett, Bruce Brombacher, Thomas Butts, John Cocharo, Barbara Currier, Steven Davis, Melissa Desjarlais, Steve Dunbar, Sister Josanne Furey, Michele Ghrist, Peter Gilchrist, Dick Gibbs, Jerrold Grossman, Joel Haack, Margie Raub Hunt, J. Alfredo Jimenez, Elgin Johnston, Dan Kennedy, Joe Kennedy, Geneivieve M. Knight, Gerald A. Krause, Sheila Krilov, Norb Kuenzi, Yo Ju Kuo, Sylvia Lazarnick, Bonnie Leitch, Victor Levine, Yung-Way Liu, Jeff Misener, Ellen Panofsky, Harry Sedinger, J. Sriskandarajah, David Torney, David Wells, LeRoy Wenstrom, and Ron Yannone.

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