THE MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

59th Annual American Mathematics Contest 12

AMC 12
CONTEST A

Solutions Pamphlet

Tuesday, FEBRUARY 12, 2008

This Pamphlet gives at least one solution for each problem on this year's contest and shows that all problems can be solved without the use of a calculator. When more than one solution is provided, this is done to illustrate a significant contrast in methods, e.g., algebraic *vs* geometric, computational *vs* conceptual, elementary *vs* advanced. These solutions are by no means the only ones possible, nor are they superior to others the reader may devise.

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1. **Answer (D):** The machine worked for 2 hours and 40 minutes, or 160 minutes, to complete one third of the job, so the entire job will take $3\cdot160 = 480$ minutes, or 8 hours. Hence the doughnut machine will complete the job at 4:30 PM.

2. **Answer (A):** Note that
$$\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}.$$
The reciprocal of $\frac{7}{6}$ is $\frac{6}{7}$.

3. **Answer (C):** Note that $\frac{2}{3}$ of 10 bananas is $\frac{20}{3}$ bananas, which are worth as much as 8 oranges. So one banana is worth as much as $8\cdot\frac{3}{20}=\frac{6}{5}$ oranges. Therefore $\frac{1}{2}$ of 5 bananas are worth as much as $\frac{5}{2}\cdot\frac{6}{5}=3$ oranges.

4. **Answer (B):** Because each denominator except the first can be canceled with the previous numerator, the product is $\frac{2008}{4}=502$.

5. **Answer (B):** Because
$$\frac{2x}{3}-\frac{x}{6}=\frac{x}{2}$$
is an integer, x must be even. The case $x=4$ shows that x is not necessarily a multiple of 3 and that none of the other statements must be true.

6. **Answer (A):** Let x denote the sticker price, in dollars. Heather pays $0.85x-90$ dollars at store A and would have paid $0.75x$ dollars at store B. Thus the sticker price x satisfies $0.85x-90=0.75x-15$, so $x=750$.

7. **Answer (D):** At the rate of 4 miles per hour, Steve can row a mile in 15 minutes. During that time $15\cdot10=150$ gallons of water will enter the boat. LeRoy must bail $150-30=120$ gallons of water during that time. So he must bail at the rate of at least $\frac{120}{15}=8$ gallons per minute.

OR

Steve must row for 15 minutes to reach the shore, so the amount of water in the boat can increase by at most $\frac{30}{15}=2$ gallons per minute. Therefore LeRoy must bail out at least $10-2=8$ gallons per minute.

8. **Answer (C):** Let x be the side length of the larger cube. The larger cube has surface area $6x^2$, and the smaller cube has surface area 6. So $6x^2=2\cdot6=12$, and $x=\sqrt{2}$. The volume of the larger cube is $x^3=\left(\sqrt{2}\right)^3=2\sqrt{2}$.

9. **Answer (D):** Let h and w be the height and width of the screen, respectively, in inches. By the Pythagorean Theorem, $h{:}w{:}27 = 3{:}4{:}5$, so

$$h = \frac{3}{5} \cdot 27 = 16.2 \quad \text{and} \quad w = \frac{4}{5} \cdot 27 = 21.6.$$

The height of the non-darkened portion of the screen is half the width, or 10.8 inches. Therefore the height of each darkened strip is

$$\frac{1}{2}(16.2 - 10.8) = 2.7 \quad \text{inches.}$$

OR

The screen has dimensions $4a \times 3a$ for some a. The portion of the screen not covered by the darkened strips has aspect ratio 2:1, so it has dimensions $4a \times 2a$. Thus the darkened strips each have height $\frac{a}{2}$. By the Pythagorean Theorem, the diagonal of the screen is $5a = 27$ inches. Hence the height of each darkened strip is $\frac{27}{10} = 2.7$ inches.

10. **Answer (D):** In one hour Doug can paint $\frac{1}{5}$ of the room, and Dave can paint $\frac{1}{7}$ of the room. Working together, they can paint $\frac{1}{5} + \frac{1}{7}$ of the room in one hour. It takes them t hours to do the job, but because they take an hour for lunch, they work for only $t - 1$ hours. The fraction of the room that they paint in this time is

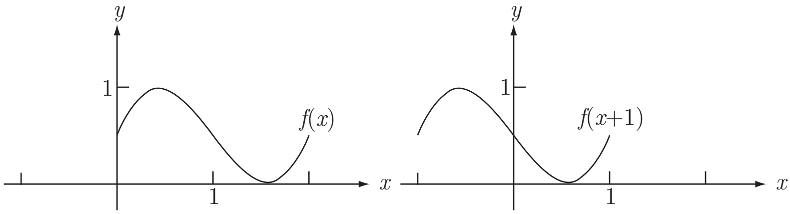
$$\left(\frac{1}{5} + \frac{1}{7}\right)(t - 1),$$

which must be equal to 1. It may be checked that the solution, $t = \frac{47}{12}$, does not satisfy the equation in any of the other answer choices.

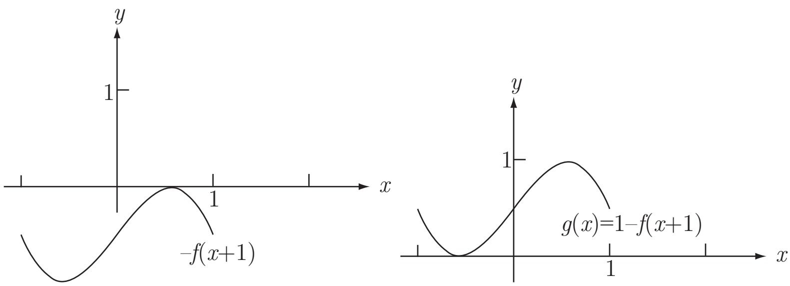
11. **Answer (C):** The sum of the six numbers on each cube is $1+2+4+8+16+32 = 63$. The three pairs of opposite faces have numbers with sums $1 + 32 = 33$, $2 + 16 = 18$, and $4 + 8 = 12$. On the two lower cubes, the numbers on the four visible faces have the greatest sum when the 4 and the 8 are hidden. On the top cube, the numbers on the five visible faces have the greatest sum when the 1 is hidden. Thus the greatest possible sum is $3 \cdot 63 - 2 \cdot (4 + 8) - 1 = 164$.

12. **Answer (B):** Because the domain of f is $[0, 2]$, $f(x + 1)$ is defined for $0 \le x + 1 \le 2$, or $-1 \le x \le 1$. Thus $g(x)$ is also defined for $-1 \le x \le 1$, so its domain is $[-1, 1]$. Because the range of f is $[0, 1]$, the values of $f(x + 1)$ are all the numbers between 0 and 1, inclusive. Thus the values of $g(x)$ are all the numbers between $1 - 0 = 1$ and $1 - 1 = 0$, inclusive, so the range of g is $[0, 1]$.

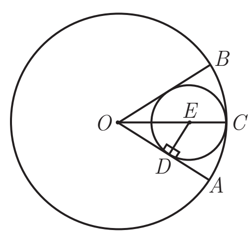
OR



The graph of $y = f(x+1)$ is obtained by shifting the graph of $y = f(x)$ one unit to the left. The graph of $y = -f(x+1)$ is obtained by reflecting the graph of $y = f(x+1)$ across the x-axis. The graph of $y = g(x) = 1 - f(x+1)$ is obtained by shifting the graph of $y = -f(x+1)$ up one unit. As the figures illustrate, the domain and range of g are $[-1, 1]$ and $[0, 1]$, respectively.



13. **Answer (B):** Let r and R be the radii of the smaller and larger circles, respectively. Let E be the center of the smaller circle, let $\overline{OC}$ be the radius of the larger circle that contains E, and let D be the point of tangency of the smaller circle to $\overline{OA}$. Then $OE = R - r$, and because $\triangle EDO$ is a $30-60-90^\circ$ triangle, $OE = 2DE = 2r$. Thus $2r = R - r$, so $\frac{r}{R} = \frac{1}{3}$. The ratio of the areas is $(\frac{1}{3})^2 = \frac{1}{9}$.



14. **Answer (A):** The boundaries of the region are the two pairs of parallel lines

$$(3x - 18) + (2y + 7) = \pm 3 \quad \text{and} \quad (3x - 18) - (2y + 7) = \pm 3.$$

These lines intersect at $(6, -2)$, $(6, -5)$, $(5, -\frac{7}{2})$, and $(7, -\frac{7}{2})$. Thus the region is a rhombus whose diagonals have lengths 2 and 3. The area of the rhombus is half the product of the diagonal lengths, which is 3.

15. **Answer (D):** The units digit of 2^n is 2, 4, 8, and 6 for $n = 1$, 2, 3, and 4, respectively. For $n > 4$, the units digit of 2^n is equal to that of 2^{n-4}. Thus for every positive integer j the units digit of 2^{4j} is 6, and hence 2^{2008} has a units digit of 6. The units digit of 2008^2 is 4. Therefore the units digit of k is 0, so the units digit of k^2 is also 0. Because 2008 is even, both 2008^2 and 2^{2008} are multiples of 4. Therefore k is a multiple of 4, so the units digit of 2^k is 6, and the units digit of $k^2 + 2^k$ is also 6.

16. **Answer (D):** The first three terms of the sequence can be written as $3\log a + 7\log b$, $5\log a + 12\log b$, and $8\log a + 15\log b$. The difference between consecutive terms can be written either as

$$(5\log a + 12\log b) - (3\log a + 7\log b) = 2\log a + 5\log b$$

or as

$$(8\log a + 15\log b) - (5\log a + 12\log b) = 3\log a + 3\log b.$$

Thus $\log a = 2\log b$, so the first term of the sequence is $13\log b$, and the difference between consecutive terms is $9\log b$. Hence the 12^{th} term is

$$(13 + (12-1)\cdot 9)\log b = 112\log b = \log(b^{112}).$$

17. **Answer (D):** If a_1 is even, then $a_2 = (a_1/2) < a_1$, so the required condition is not met. If $a_1 \equiv 1 \pmod 4$, then $a_2 = 3a_1 + 1$ is a multiple of 4, so $a_3 = (3a_1+1)/2$, and $a_4 = (3a_1+1)/4 \le a_1$. Hence the required condition is also not met in this case. If $a_1 \equiv 3 \pmod 4$, then a_2 is even but not a multiple of 4. It follows that $a_3 = (3a_1+1)/2 > a_1$, and a_3 is odd, so $a_4 = 3a_3 + 1 > a_3 > a_1$. Because 2008 is a multiple of 4, a total of $\frac{2008}{4} = 502$ possible values of a_1 are congruent to $3 \pmod 4$. These 502 values of a_1 meet the required condition.

Note: It is a famous unsolved problem to show whether or not the number 1 must be a term of this sequence for every choice of a_1.

18. **Answer (C):** It may be assumed that $A = (a,0,0)$, $B = (0,b,0)$, $C = (0,0,c)$, $AB = 5$, $BC = 6$, and $CA = 7$. Then

$$a^2 + b^2 = 5^2, \quad b^2 + c^2 = 6^2, \quad \text{and} \quad a^2 + c^2 = 7^2,$$

from which

$$a^2 + b^2 + c^2 = \frac{1}{2}\left(5^2 + 6^2 + 7^2\right) = 55.$$

It follows that $a = \sqrt{55 - 6^2} = \sqrt{19}$, $b = \sqrt{55 - 7^2} = \sqrt{6}$, $c = \sqrt{55 - 5^2} = \sqrt{30}$, and the volume of tetrahedron $OABC$ can be expressed as

$$\frac{1}{3}\cdot OC \cdot \text{Area}(\triangle OAB) = \frac{1}{6}\sqrt{6\cdot 19\cdot 30} = \sqrt{95}.$$

19. **Answer (C):** Each term in the expansion has the form x^{a+b+c}, where $0 \le a \le 27$, $0 \le b \le 14$, and $0 \le c \le 14$. There are $(14+1)^2 = 225$ possible combinations of values for b and c, and for every combination except $(b, c) = (0, 0)$, there is a unique a with $a + b + c = 28$. Thus the coefficient of x^{28} is 224.

OR

Let $P(x) = (1 + x + x^2 + \cdots + x^{14})^2 = 1 + r_1x + r_2x^2 + \cdots + r_{28}x^{28}$ and $Q(x) = 1 + x + x^2 + \cdots + x^{27}$. The coefficient of x^{28} in the product $P(x)Q(x)$ is $r_1 + r_2 + \cdots + r_{28} = P(1) - 1 = 15^2 - 1 = 224$.

20. **Answer (E):** By the Angle Bisector Theorem,

$$AD = 5 \cdot \frac{3}{3+4} = \frac{15}{7} \quad \text{and} \quad BD = 5 \cdot \frac{4}{3+4} = \frac{20}{7}.$$

To determine CD, start with the relation Area($\triangle ADC$) + Area($\triangle BCD$) = Area($\triangle ABC$) to get

$$\frac{3 \cdot CD}{2\sqrt{2}} + \frac{4 \cdot CD}{2\sqrt{2}} = \frac{3 \cdot 4}{2}.$$

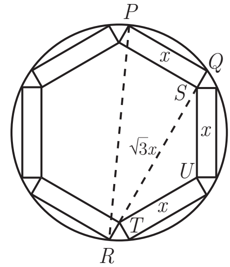
This gives $CD = \frac{12\sqrt{2}}{7}$. Now use the fact that the area of a triangle is given by rs, where r is the radius of the inscribed circle and s is half the perimeter of the triangle. The ratio of the area of $\triangle ADC$ to the area of $\triangle BCD$ is the ratio of the altitudes to their common base $\overline{CD}$, which is $\frac{AD}{BD} = \frac{3}{4}$. Hence

$$\frac{3}{4} = \frac{\text{Area}(\triangle ADC)}{\text{Area}(\triangle BCD)} = \frac{r_a(3 + \frac{15}{7} + \frac{12\sqrt{2}}{7})}{r_b(4 + \frac{20}{7} + \frac{12\sqrt{2}}{7})}.$$

which yields

$$\frac{r_a}{r_b} = \frac{3(4+\sqrt{2})}{4(3+\sqrt{2})} = \frac{3}{28}(10 - \sqrt{2}).$$

21. **Answer (D):** Call a permutation <u>balanced</u> if $a_1 + a_2 = a_4 + a_5$, and consider the number of balanced permutations. The sum of all five entries is odd, so in a balanced permutation, a_3 must be 1, 3, or 5. For each choice of a_3, there is a unique way to group the remaining four numbers into two sets whose elements have equal sums. For example, if $a_3 = 1$, the two sets must be $\{2, 5\}$ and $\{3, 4\}$. Any one of the four numbers can be a_1, and the value of a_2 is then determined. Either of the two remaining numbers can be a_4, and the value of a_5 is then determined. Thus there are $3 \cdot 2 \cdot 4 = 24$ balanced permutations of $(1, 2, 3, 4, 5)$, and $5! - 24 = 96$ permutations that are not balanced. Call a permutation <u>heavy-headed</u> if $a_1 + a_2 > a_4 + a_5$. Reversing the entries in a heavy-headed permutation yields a unique heavy-tailed permutation, and vice versa, so there are exactly as many heavy-headed permutations as heavy-tailed ones. Therefore the number of heavy-tailed permutations is $\frac{1}{2} \cdot 96 = 48$.



22. **Answer (C):** Select one of the mats. Let P and Q be the two corners of the mat that are on the edge of the table, and let R be the point on the edge of the table that is diametrically opposite P as shown. Then R is also a corner of a mat and $\triangle PQR$ is a right triangle with hypotenuse $PR = 8$. Let S be the inner corner of the chosen mat that lies on $\overline{QR}$, T the analogous point on the mat with corner R, and U the corner common to the other mat with corner S and the other mat with corner T. Then $\triangle STU$ is an isosceles triangle with two sides of length x and vertex angle $120°$. It follows that $ST = \sqrt{3}x$, so $QR = QS+ST+TR = \sqrt{3}x+2$. Apply the Pythagorean Theorem to $\triangle PQR$ to obtain $\left(\sqrt{3}x+2\right)^2 + x^2 = 8^2$, from which $x^2 + \sqrt{3}x - 15 = 0$. Solve for x and ignore the negative root to obtain

$$x = \frac{3\sqrt{7}-\sqrt{3}}{2}.$$

23. **Answer (D):** Adding $1+i$ to each side of the given equation gives

$$1+i = (z^4+4z^3i-6z^2-4zi-i)+1+i = z^4+4z^3i-6z^2-4zi+1 = (z+i)^4.$$

Let $w = z+i = r(\cos\theta + i\sin\theta)$. Since

$$i+1 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right),$$

the solutions of $w^4 = 1+i$ satisfy

$$r^4 = \sqrt{2} \quad \text{and} \quad \theta = \frac{1}{4}\left(\frac{\pi}{4}+2k\pi\right) = \frac{\pi}{16}+\frac{\pi}{2}k,$$

for $k = 0$, 1, 2, or 3. Thus

$$w_k = 2^{1/8}\left(\cos\left(\frac{\pi}{16}+\frac{\pi}{2}k\right) + i\sin\left(\frac{\pi}{16}+\frac{\pi}{2}k\right)\right) \quad \text{for } k = 0,1,2, \text{ or } 3,$$

and the four solutions for $z = w - i$ are

$$z_k = 2^{1/8}\left(\cos\left(\frac{\pi}{16}+\frac{\pi}{2}k\right) + i\sin\left(\frac{\pi}{16}+\frac{\pi}{2}k\right)\right) - i \quad \text{for } k = 0,1,2, \text{ or } 3.$$

Note that w_0, w_1, w_2, and w_3 are equally spaced around the circle of radius $2^{1/8}$ centered at $(0,0)$, so z_0, z_1, z_2, and z_3 are equally spaced around the circle of radius $2^{1/8}$ centered at $(0,-1)$. Therefore z_0, z_1, z_2, and z_3 are vertices of a square with side length $2^{1/8}\sqrt{2} = 2^{5/8}$ and area $(2^{5/8})^2 = 2^{5/4}$.

OR

The Binomial Theorem gives

$$(z+i)^4 = z^4+4z^3i-6z^2-4zi+1 = (z^4+4z^3i-6z^2-4zi-i)+1+i = 1+i.$$

Let a satisfy $a^4 = 1+i$, and let $w = z+i$. Then $w^4 = a^4$, so the possible values for w are a, ia, $-a$, and $-ia$, which are the vertices of a square with diagonal $2|a| = 2\sqrt[8]{2}$. The transformation $w = z+i$ is a translation, so it preserves area. Hence the area of the original polygon is $(2\sqrt[8]{2})^2/2 = 2\sqrt[4]{2} = 2^{5/4}$.

24. **Answer (D):** Let $C = (0,0)$, $B = (2, 2\sqrt{3})$, and $A = (x, 0)$ with $x > 0$. Then $D = (1, \sqrt{3})$. Let P be on the positive x-axis to the right of A. Then $\angle BAD = \angle PAD - \angle PAB$. Provided $\angle PAD$ and $\angle PAB$ are not right angles, it follows that

$$\tan(\angle BAD) = \tan(\angle PAD - \angle PAB) = \frac{\tan(\angle PAD) - \tan(\angle PAB)}{1 + \tan(\angle PAD)\tan(\angle PAB)}$$
$$= \frac{m_{AD} - m_{AB}}{1 + m_{AD}m_{AB}} = \frac{\frac{\sqrt{3}}{1-x} - \frac{2\sqrt{3}}{2-x}}{1 + \frac{\sqrt{3}}{1-x}\cdot\frac{2\sqrt{3}}{2-x}} = \frac{\sqrt{3}x}{x^2 - 3x + 8}$$
$$= \frac{\sqrt{3}}{\left(\sqrt{x} - \frac{2\sqrt{2}}{\sqrt{x}}\right)^2 + (4\sqrt{2} - 3)} \le \frac{\sqrt{3}}{4\sqrt{2} - 3},$$

with equality when $x = 2\sqrt{2}$. If $\angle PAD = 90^\circ$, then

$$\tan(\angle BAD) = \cot(\angle PAB) = \frac{1}{2\sqrt{3}} < \frac{\sqrt{3}}{4\sqrt{2} - 3}.$$

If $\angle PAB = 90^\circ$, then

$$\tan(\angle BAD) = -\cot(\angle PAD) = \frac{1}{\sqrt{3}} < \frac{\sqrt{3}}{4\sqrt{2} - 3}.$$

Therefore the largest possible value of $\tan(\angle BAD)$ is $\sqrt{3}/(4\sqrt{2} - 3)$.

OR

Because the circle with diameter $\overline{BD}$ does not intersect the line AC, it follows that $\angle BAD < 90^\circ$. Thus the value of $\tan(\angle BAD)$ is greatest when $\angle BAD$ is greatest. This occurs when A is placed to minimize the size of the circle passing through A, B, and D, so the maximum is attained when that circle is tangent to $\overline{AC}$ at A. For this location of A, the Power of a Point Theorem implies that

$$AC^2 = CB \cdot CD = 4 \cdot 2 = 8, \text{ and } AC = \sqrt{8} = 2\sqrt{2}.$$

Because $\frac{CA}{CB} = \frac{CD}{CA}$, it follows that $\triangle CAD$ is similar to $\triangle CBA$. Thus $AB = \sqrt{2}AD$. The Law of Cosines, applied to $\triangle ADC$, gives

$$AD^2 = CD^2 + CA^2 - 2CD \cdot CA \cdot \cos 60^\circ = 12 - 4\sqrt{2}.$$

Let O be the center of the circle passing through A, B, and D. The Extended Law of Sines, applied to $\triangle ABD$ and $\triangle ADC$, gives

$$2OB = \frac{AB}{\sin(\angle BDA)} = \frac{AB}{\sin(\angle ADC)}$$

$$= \frac{AB \cdot AD}{AC \cdot \sin 60^\circ} = \frac{2AB \cdot AD}{\sqrt{3}AC}$$

$$= \frac{2\sqrt{2}AD^2}{2\sqrt{2}\sqrt{3}} = \frac{AD^2}{\sqrt{3}}.$$

Let M be the midpoint of BD. Because $\angle BAD = \frac{1}{2}\angle BOD = \angle BOM$, it follows that

$$\begin{aligned}\tan(\angle BAD) = \tan(\angle BOM) &= \frac{MB}{OM} \\ &= \frac{1}{\sqrt{OB^2 - 1}} = \frac{1}{\sqrt{\frac{AD^4}{12} - 1}} \\ &= \frac{\sqrt{3}}{\sqrt{(6-2\sqrt{2})^2 - 3}} = \frac{\sqrt{3}}{4\sqrt{2} - 3}.\end{aligned}$$

25. **Answer (D):** Let $z_n = a_n + b_n i$. Then

$$\begin{aligned}z_{n+1} &= (\sqrt{3}a_n - b_n) + (\sqrt{3}b_n + a_n)i = (a_n + b_n i)(\sqrt{3} + i) \\ &= z_n(\sqrt{3} + i) = z_1(\sqrt{3} + i)^n.\end{aligned}$$

Noting that $\sqrt{3} + i = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ and applying DeMoivre's formula gives

$$\begin{aligned}2 + 4i = z_{100} &= z_1\left(2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)\right)^{99} \\ &= z_1 \cdot 2^{99}\left(\cos\left(\frac{99\pi}{6}\right) + i\sin\left(\frac{99\pi}{6}\right)\right) \\ &= (a_1 + b_1 i) \cdot 2^{99} \cdot i = -2^{99}b_1 + 2^{99}a_1 i.\end{aligned}$$

So $2 = -2^{99}b_1$, $4 = 2^{99}a_1$, and

$$a_1 + b_1 = \frac{4}{2^{99}} - \frac{2}{2^{99}} = \frac{1}{2^{98}}.$$

OR

Note that

$$
\begin{aligned}
(a_{n+2}, b_{n+2}) &= \Big(\sqrt{3}(\sqrt{3}a_n - b_n) - (\sqrt{3}b_n + a_n), \\
&\quad \sqrt{3}(\sqrt{3}b_n + a_n) + (\sqrt{3}a_n - b_n)\Big) \\
&= \left(-2\sqrt{3}b_n + 2a_n, 2\sqrt{3}a_n + 2b_n\right), \\
(a_{n+3}, b_{n+3}) &= \Big(\sqrt{3}(-2\sqrt{3}b_n + 2a_n) - (2\sqrt{3}a_n + 2b_n), \\
&\quad \sqrt{3}(2\sqrt{3}a_n + 2b_n) + (-2\sqrt{3}b_n + 2a_n)\Big) \\
&= 8(-b_n, a_n),
\end{aligned}
$$

and $(a_{n+6}, b_{n+6}) = 8(-b_{n+3}, a_{n+3}) = -64(a_n, b_n)$. Because $97 = 1 + 16 \cdot 6$, we have

$$(a_{97}, b_{97}) = (-64)^{16}(a_1, b_1) = 2^{96}(a_1, b_1)$$

and

$$(2, 4) = (a_{100}, b_{100}) = 2^3\,(-b_{97}, a_{97}) = 2^{99}(-b_1, a_1).$$

The conclusion follows as in the first solution.

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